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On $N = 2$ superfield for $N = 2$ vector supermultiplet in two dimensional spacetime

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Abstract

We focus on the superfield formulation for a $N = 2$ vector supermultiplet in two dimensional spacetime and explicitly show that the Wess-Zumino gauge condition for a $N = 2$ superfield is compatible with familiar SUSY (plus $U(1)$ gauge) transformations for the vector supermultiplet. $N = 2$ SUSY invariant mass and Yukawa interaction terms for the vector supermultiplet are also constructed from the superfield explicitly in addition to a free (kinetic) action.

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Superfield formulation in superspace gives systematic understandings for various supersymmetric (SUSY) field theories (for further references on superfields, see [1, 2]). In two dimensional spacetime ($d = 2$), superfields were constructed in the context of discussing classical solutions for SUSY field theories [3], e.g. $N = 1$ and $N = 2$ superfields for vector supermultiplets are given as a spinor and a scalar, respectively. Moreover, a realistic SUSY model can be constructed by $N \geq 2$ SUSY in the SGM scenario [4], where we have found [5, 6] the relation (equivalence) between a non-linear (NL) SUSY model [7] and linear SUSY interacting field theories in $d = 2$. In order to study systematics in the linearization of $N \geq 2$ NLSUSY (for SUSY interacting field theories), it is important to know the details of $N \geq 2$ superfield formulation.

In this letter, we focus on the superfield formulation for the $N = 2$ vector supermultiplet in $d = 2$ and discuss on the basics in detail. Based on the $d = 2$, $N = 2$ superfield [3], we explicitly show that the Wess-Zumino (WZ) gauge condition is compatible with familiar SUSY (plus $U(1)$ gauge) transformations for the vector supermultiplet. $N = 2$ SUSY invariant mass and Yukawa interaction terms for the vector supermultiplet are also constructed from the superfield explicitly in addition to a free (kinetic) action.

Let us first introduce the following $N = 2$ superfield in $d = 2$ [3] on superspace coordinates (x^a, θ_α^i) ,[‡]

$$\begin{aligned} \mathcal{V}(x, \theta^i) = & C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x) \\ & - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2} \bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8} \bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \end{aligned} \quad (1)$$

where the component fields are denoted by (C, D) for two scalar fields, (Λ^i, λ^i) for four spinor fields, ϕ for a pseudo scalar field, v^a for a vector field, and $M^{ij} = M^{(ij)}$ ($= \frac{1}{2}(M^{ij} + M^{ji})$) for three scalar fields ($M^{ii} = \delta^{ij} M^{ij}$), respectively. Under superspace translations,

$$x'^a = x^a + i \bar{\zeta}^i \gamma^a \theta^i, \quad \theta'_\alpha{}^i = \theta_\alpha^i + \zeta_\alpha^i, \quad (2)$$

the superfield (1) transforms as

$$\delta_\zeta \mathcal{V}(x, \theta^i) = \bar{\zeta}^i Q^i \mathcal{V}(x, \theta^i) \quad (3)$$

[‡]Minkowski spacetime indices in $d = 2$ are denoted by $a, b, \dots = 0, 1$ and $SO(2)$ internal indices are $i, j, \dots = 1, 2$. The Minkowski spacetime metric is $\frac{1}{2}\{\gamma^a, \gamma^b\} = \eta^{ab} = \text{diag}(+, -)$. As for the conventions in $d = 2$ for the γ matrices etc., for example, see [8].

with supercharges

$$Q_\alpha^i = \frac{\partial}{\partial \theta^{\alpha i}} + i \not{\partial} \theta_\alpha^i, \quad (4)$$

satisfying $\{Q_\alpha^i, Q_\beta^j\} = -2\delta^{ij}(\gamma^a C)_{\alpha\beta} P_a$. The superfield transformation (3) gives SUSY transformations for the component fields as,

$$\begin{aligned} \delta_\zeta C &= \bar{\zeta}^i \Lambda^i, \\ \delta_\zeta \Lambda^i &= -i \not{\partial} C \zeta^i + M^{ij} \zeta^j - M^{jj} \zeta^i + \frac{1}{2} \epsilon^{ij} \phi \gamma_5 \zeta^j - \frac{i}{2} \epsilon^{ij} v \cdot \gamma \zeta^j, \\ \delta_\zeta M^{12} &= \bar{\zeta}^{(1} \lambda^{2)} - i \bar{\zeta}^{(1} \not{\partial} \Lambda^{2)}, \\ \delta_\zeta M^{11} &= \bar{\zeta}^1 \lambda^1 + i \bar{\zeta}^2 \not{\partial} \Lambda^2, \quad \delta_\zeta M^{22} = \bar{\zeta}^2 \lambda^2 + i \bar{\zeta}^1 \not{\partial} \Lambda^1, \\ \delta_\zeta \phi &= \epsilon^{ij} (-\bar{\zeta}^i \gamma_5 \lambda^j - i \bar{\zeta}^i \gamma_5 \not{\partial} \Lambda^j), \\ \delta_\zeta v^a &= \epsilon^{ij} (-i \bar{\zeta}^i \gamma^a \lambda^j - \bar{\zeta}^i \not{\partial} \gamma^a \Lambda^j), \\ \delta_\zeta \lambda^i &= -i \not{\partial} M^{ij} \zeta^j - \frac{i}{2} \epsilon^{ij} \gamma_5 \not{\partial} \phi \zeta^j - \frac{1}{2} \epsilon^{ij} \gamma_a \not{\partial} v^a \zeta^j + D \zeta^i, \\ \delta_\zeta D &= -i \bar{\zeta}^i \not{\partial} \lambda^i. \end{aligned} \quad (5)$$

Next following the ref.[1] we define the SUSY generalization of a gauge transformation of $\mathcal{V}(x, \theta^i)$ as

$$\delta_g \mathcal{V} = \Phi^1 + \alpha \Phi^2, \quad (6)$$

where α is an arbitrary real parameter, and Φ^i are generalized gauge parameters in the form of $N = 2$ scalar superfields for a $N = 2$ scalar supermultiplet [8],

$$\begin{aligned} \Phi^i(x, \theta^i) &= B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij} \bar{\theta}^j \nu(x) - \frac{1}{2} \bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i \bar{\theta}^i \not{\partial} B^j(x) \theta^j \\ &\quad + \frac{i}{2} \bar{\theta}^j \theta^j (\bar{\theta}^i \not{\partial} \chi(x) - \epsilon^{ik} \bar{\theta}^k \not{\partial} \nu(x)) + \frac{1}{8} \bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \square B^i(x) \end{aligned} \quad (7)$$

with the component fields being denoted by B^i for two scalar fields, (χ, ν) for two spinor fields and F^i for two auxiliary scalar fields. (The gauge transformation (6) may be recasted by using the ordinary (covariant) derivative as for the spinor-superfield case [3].) The explicit component form of the gauge transformation (6) is

$$\begin{aligned} \delta_g C &= B^1 + \alpha B^2, \\ \delta_g \Lambda^1 &= \chi + \alpha \nu, \quad \delta_g \Lambda^2 = \alpha \chi - \nu, \\ \delta_g M^{12} &= \alpha F^1 + F^2, \\ \delta_g M^{11} &= F^1 - \alpha F^2, \quad \delta_g M^{22} = -F^1 + \alpha F^2, \end{aligned}$$

$$\begin{aligned}
\delta_g \phi &= 0, \\
\delta_g v^a &= -2\partial^a(\alpha B^1 - B^2), \\
\delta_g \lambda^1 &= -i\not{\partial}(\chi + \alpha\nu), \quad \delta_g \lambda^2 = -i\not{\partial}(\alpha\chi - \nu), \\
\delta_g D &= -\square(B^1 + \alpha B^2).
\end{aligned} \tag{8}$$

Note that v^a transforms as an Abelian gauge field and, in addition to ϕ , M^{ii} is a gauge invariant quantity ($\delta_g M^{ii} = 0$). By using the gauge freedoms of (B^i, χ, ν, F^i) in Eq.(8), we take the WZ gauge [3],

$$C = 0, \quad \Lambda^i = 0, \quad M^{12} = 0, \quad M^{11} - M^{22} = 0, \tag{9}$$

and the superfield (1) is rewritten in terms of $A = M^{ii}$. Then, the generalized gauge transformations (8) become the ordinary ones,

$$\begin{aligned}
\delta_g A &= \delta_g M^{ii} = 0, \\
\delta_g \phi &= 0, \\
\delta_g v^a &= -2\partial^a(\alpha B^1 - B^2), \\
\delta_g \lambda^i &= 0, \\
\delta_g D &= 0,
\end{aligned} \tag{10}$$

where the gauge parameters of v^a are essentially one parameter because one of them is used by $C = 0$ in Eq.(9). The superfield (1) also reduces to [9]

$$\begin{aligned}
V(x, \theta^i) &= \mathcal{V}(x, \theta^i)|_{\text{WZ gauge}} \\
&= -\frac{1}{4}\bar{\theta}^i \theta^i A(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i \gamma_5 \theta^j \phi(x) - \frac{i}{4}\epsilon^{ij}\bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) \\
&\quad - \frac{1}{8}\bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x).
\end{aligned} \tag{11}$$

The WZ gauge (9) is violated by the SUSY transformations (5). Therefore, we consider modified SUSY transformations (for example, see [10]),

$$\tilde{\delta}_\zeta = \delta_\zeta + \delta_g. \tag{12}$$

in order to maintain the WZ gauge condition under Eq.(12). For the eliminated component fields by Eq.(9), the modified SUSY transformations (12) are written as

$$\tilde{\delta}_\zeta C = B^1 + \alpha B^2,$$

$$\begin{aligned}
\tilde{\delta}_\zeta \Lambda^1 &= -A\zeta^1 + M^{11}\zeta^1 + \frac{1}{2}\phi\gamma_5\zeta^2 - \frac{i}{2}v \cdot \gamma\zeta^2 + \chi + \alpha\nu, \\
\tilde{\delta}_\zeta \Lambda^2 &= -A\zeta^2 + M^{22}\zeta^2 - \frac{1}{2}\phi\gamma_5\zeta^1 + \frac{i}{2}v \cdot \gamma\zeta^1 + \alpha\chi - \nu, \\
\tilde{\delta}_\zeta M^{12} &= \bar{\zeta}^{(1}\lambda^{2)} + \alpha F^1 + F^2, \\
\tilde{\delta}_\zeta (M^{11} - M^{22}) &= \bar{\zeta}^1\lambda^1 - \bar{\zeta}^2\lambda^2 + 2(F^1 - \alpha F^2),
\end{aligned} \tag{13}$$

and we impose

$$\tilde{\delta}_\zeta C = 0, \quad \tilde{\delta}_\zeta \Lambda^i = 0, \quad \tilde{\delta}_\zeta M^{12} = 0, \quad \tilde{\delta}_\zeta (M^{11} - M^{22}) = 0, \tag{14}$$

i.e.

$$\begin{aligned}
B^1 + \alpha B^2 &= 0, \\
\chi + \alpha\nu &= A\zeta^1 - M^{11}\zeta^1 - \frac{1}{2}\phi\gamma_5\zeta^2 + \frac{i}{2}v \cdot \gamma\zeta^2, \\
\alpha\chi - \nu &= A\zeta^2 - M^{22}\zeta^2 + \frac{1}{2}\phi\gamma_5\zeta^1 - \frac{i}{2}v \cdot \gamma\zeta^1, \\
\alpha F^1 + F^2 &= -\bar{\zeta}^{(1}\lambda^{2)}, \\
F^1 - \alpha F^2 &= -\frac{1}{2}(\bar{\zeta}^1\lambda^1 - \bar{\zeta}^2\lambda^2).
\end{aligned} \tag{15}$$

Substituting these (in particular, the first three equations in Eq.(15)) into Eq.(12) for the remaining component fields (v^a , λ^i , A , ϕ , D) gives the familiar SUSY transformations for the $d = 2$, $N = 2$ vector supermultiplet [5] plus the $U(1)$ gauge transformation of v^a ,

$$\begin{aligned}
\tilde{\delta}_\zeta A \ (= \ \tilde{\delta}_\zeta M^{ii}) &= \bar{\zeta}^i\lambda^i, \\
\tilde{\delta}_\zeta \phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
\tilde{\delta}_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j - 2\partial^a(\alpha B^1 - B^2), \\
\tilde{\delta}_\zeta \lambda^i &= (D - i\not{\partial}A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\not{\partial}\phi\zeta^j, \\
\tilde{\delta}_\zeta D &= -i\bar{\zeta}^i\not{\partial}\lambda^i,
\end{aligned} \tag{16}$$

where $F_{ab} = \partial_a v_b - \partial_b v_a$, and $-2(\alpha B^1 - B^2)$ is the gauge parameter for the $U(1)$ gauge. The SUSY transformations (16) satisfy a closed commutator algebra

$$[\tilde{\delta}_{\zeta_1}, \tilde{\delta}_{\zeta_2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{17}$$

where $\delta_P(\Xi^a)$ means a translation with a generator $\Xi^a = 2i\bar{\zeta}_1^i\gamma^a\zeta_2^i$, and $\delta_g(\theta)$ is the $U(1)$ gauge transformation with a generator $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$,

which appears in the commutator for v^a . As is shown below, actions (Eqs. from (20) to (22)) constructed from the superfield (11) are also SUSY and $U(1)$ -gauge invariant under Eq.(16). Therefore, the superfield (1) reduces to Eq.(11) in the WZ gauge (9) with the well-defined SUSY and $U(1)$ gauge transformations (16) without violating the valance between the bosonic and fermionic degrees of freedom. In the above arguments, when $\alpha = 1$ the symmetrization of the indices for M^{12} is manifest, while when $\alpha = \pm i$ the $U(1)$ gauge transformation is not induced in Eq.(16).

Let us evaluate SUSY invariant (renormalizable) actions for the $N = 2$ vector supermultiplet [5], which are constructed from the superfield (11) with the component fields $(v^a, \lambda^i, A, \phi, D)$. By using differential operators in $N = 2$ superspace,

$$D_\alpha^i = \frac{\partial}{\partial \theta^{\alpha i}} - i \not{\partial} \theta_\alpha^i, \quad (18)$$

we define the following scalar and pseudo scalar superfields,

$$W^{ij} = \bar{D}^i D^j V, \quad W_5^{ij} = \bar{D}^i \gamma_5 D^j V. \quad (19)$$

Then, the free (kinetic) action is obtained from W^{ij} and W_5^{ij} as

$$\begin{aligned} S_{V0} &= \frac{1}{4} \int d^2x \left[\int d^2\theta^i \frac{1}{8} (\overline{D^j W^{kl}} D^j W^{kl} + \overline{D^j W_5^{kl}} D^j W_5^{kl}) + \frac{1}{4} \int d^4\theta^i \frac{8}{\kappa} \xi V \right]_{\theta^i=0} \\ &= \int d^2x \left\{ -\frac{1}{4} (F_{ab})^2 + \frac{i}{2} \bar{\lambda}^i \not{\partial} \lambda^i + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2 - \frac{1}{\kappa} \xi D \right\}, \end{aligned} \quad (20)$$

where the last term means the Fayet-Iliopoulos D -term with an arbitrary dimensionless paramater ξ . On the other hand, the quadratic and cubic terms of W^{ij} and W_5^{ij} leads to the mass and Yukawa interaction terms as follows:

$$\begin{aligned} S_{Vm} &= \frac{1}{4} \int d^2x \, m \left[\int d^2\theta^i \{ (W^{jk})^2 + (W_5^{jk})^2 \} + \int d\bar{\theta}^i d\theta^j (W^{ik} W^{jk} + W_5^{ik} W_5^{jk}) \right]_{\theta^i=0} \\ &= \int d^2x \left\{ -\frac{1}{2} m (\bar{\lambda}^i \lambda^i - 2AD + \epsilon^{ab} \phi F_{ab}) \right\}, \end{aligned} \quad (21)$$

$$\begin{aligned} S_{Vf} &= \frac{1}{4} \int d^2x \, f \left[\frac{1}{2} \int d^2\theta^i W^{jk} (W^{jl} W^{kl} + W_5^{jl} W_5^{kl}) \right. \\ &\quad \left. + \int d\bar{\theta}^i d\theta^j \{ W^{ij} (W^{kl} W^{kl} + W_5^{kl} W_5^{kl}) + W^{ik} (W^{jl} W^{kl} + W_5^{jl} W_5^{kl}) \} \right]_{\theta^i=0} \\ &= \int d^2x \{ f (A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j - A^2 D + \phi^2 D + \epsilon^{ab} A \phi F_{ab}) \}, \end{aligned} \quad (22)$$

where f is an arbitrary constant with the dimension (mass)¹. The actions (20), (21) and (22) are invariant under the $N = 2$ SUSY and $U(1)$ gauge transformations (16), respectively.

To summarize, we have shown that the superfield (1) consistently reduces to Eq.(11) with the minimal (off-shell) set of the component fields $(v^a, \lambda^i, A, \phi, D)$ in the WZ gauge (9). The condition (14), i.e. Eq.(15) to maintain the WZ gauge (9) under the modified SUSY transformations (12) gives the familiar $N = 2$ SUSY and $U(1)$ gauge transformations (16). The free (kinetic) action, the mass and Yukawa interaction terms for the $N = 2$ vector supermultiplet have been obtained from the superfield (11) as in Eqs. from (20) to (22). The extension of this $N = 2$ superfield formulation to $d = 4$ is an important problem which is now in progress. Also the coupling of matter supermultiplets to the $N = 2$ superfield (1) is an interesting problem under study.

References

- [1] J. Wess and J. Bagger, *Supersymmetry and Supergravity (Second Edition, Revised and Expanded)* (Princeton University Press, Princeton, New Jersey, 1992).
- [2] P. West, *Introduction to Supersymmetry and Supergravity (Extended Second Edition)* (World Scientific, Singapore, 1990).
- [3] P. Di Vecchia and S. Ferrara, *Nucl. Phys.* **B130** (1977) 93.
- [4] K. Shima, M. Tsuda and W. Lang, *Phys. Lett.* **B659** (2008) 741, arXiv:0710.1680 [hep-th].
- [5] K. Shima and M. Tsuda, *Mod. Phys. Lett.* **A22** (2007) 1085, arXiv:hep-th/0611051.
- [6] K. Shima and M. Tsuda, *Mod. Phys. Lett.* **A22** (2007) 3027, arXiv:0710.0063 [hep-th].
- [7] D.V. Volkov and V.P. Akulov, *Phys. Lett.* **B46** (1973) 109.
- [8] T. Uematsu and C.K. Zachos, *Nucl. Phys.* **B201** (1982) 250.
- [9] Eq.(11) is obtained from the dimensional reduction from $d = 4$, see N. Ohta, *Z. Phys.* **C24** (1984) 327.
- [10] Y. Matsumura, N. Sakai and T. Sakai, *Phys. Rev.* **D52** (1995) 2446, arXiv:hep-th/9504150.